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Estimation of Reliability in Multicomponent Stress-Strength Model Following Burr Type XII Distribution under Selective Ranked Set Sampling

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ABSTRACT
The stress-strength model for system reliability for multicomponent system when a device under consideration is a combination of $\mathcal{K}$ usually independent components with strengths $X_1, X_2, \dots, X_{\mathcal{K}}$ and each component experiencing a common stress Y. The system is regarded as alive if at least$\mathcal{S}$ out of $\mathcal{K}(\mathcal{S} < \mathcal{K})$strengths exceed the stress. The reliability of such system is obtained when the stress and strengths are assumed to have Burr XII distributions with common shape parameter(c). The research methodology adapted here is to estimate the parameters by using maximum likelihood method. Based on different types of ranked set sampling, the reliability estimators are obtained when samples drawn from strength and stress distributions. Efficiencies of reliability estimators based on ranked set sampling, median ranked set sampling, extreme ranked set sampling and percentile ranked set sampling are calculated with respect to reliability estimators based on simple random sampling procedure.
***Keywords*-**Burr type XII, reliability estimation, ranked set sampling, median ranked set sampling, extreme ranked set sampling, percentile ranked set sampling.

I. INTRODUCTION

In experimental investigations, situations may occur in which precise measurement of the characteristic under study is expensive or difficult, but elements can be readily chosen from the population of interest, and a small number can be ordered easily and fairly accurately by eye or by some other means not requiring quantification, it turns out that the use of McIntyre's (1952) notation of ranked set sampling (RSS). He introduced the basic concept behind RSS. The RSS procedure can be described as follows. Randomly select n random samples from the population of interest each of size n. The units within each sample are ranked with respect to a variable of interest by visual inspection or any cheap method. Then the smallest and second smallest units from the first and second samples are selected for actual measurement. The procedure is continued until the largest unit from the n^{th} sample is selected for measurements. Thus total of n measured units, which represented one cycle are obtained. This procedure may be repeated r times until number of nr units are obtained. These nr units form the RSS data.

McIntyre (1952) claimed that RSS procedure more accurate estimators of the sample mean than the usual simple random sampling (SRS). Takahasi and Wakimato (1968) independently described the same sampling method and presented the mathematical theory, which support McIntyre's claims. Dell and Clutter (1972) showed that the RSS mean remains unbiased and more efficient than the simple random sampling mean even if ranking is imperfect.

Various authors proposed modified ranked set sampling to get better estimators for the population mean. One of the popular schemes is to use the median ranked set sampling (MRSS) (see Bhoj, 1997; Muttlak 1997). MRSS procedure can be summarized as follows: randomly select n random samples from the population of interest each of size n. The units within each sample are ranked with respect to a variable of interest. If the sample size n is odd, select the median element of each ordered set. If the sample size n is even, select for the measurement from the first $n/2$ samples the $(n/2)^{th}$ smallest ranked unit and from the second $n/2$ samples the $(n/2+1)^{th}$ smallest ranked unit.The cycle can be repeated r times if needed to get a sample of size nr units from MRSS data.

To reduce the error in ranking, several modification of the RSS technique had been suggested. Samawi et al.(1996) investigated using extreme ranked set sampling (ERSS) to estimate the population mean. In ERSS, select n random samples of size n units from the population under consideration. Rank the units within each sample with respect to a variable of interest by visual inspection or any other cost free method. If the sample size n is odd, select from $(n-1)/2$ samples the smallest unit, from the other $(n-1)/2$ the largest unit and for one sample the median of the sample for actual measurement. If the sample size is even, select from $n/2$ samples the smallest unit and from the other $n/2$ samples the largest unit for actual measurement. The cycle may be repeated r times to get nr units from ERSS data.

Muttlak (2003) introduced the percentile ranked set sampling (PRSS) approach as a modification to RSS. The PRSS procedure can be summarized as follows: n ranked random samples each of size n are drawn from a population under consideration. If the sample size is even, select for measurement from the first $n/2$ samples the $s(n+1)^{th}$ smallest ranked unit and from the second $n/2$ samples the $t(n+1)^{th}$ smallest ranked unit, where $t = 1 - s$ and $0 < s \leq 0.5$. If the sample size is odd, select from the first $(n-1)/2$ samples the $s(n+1)^{th}$ smallest ranked unit and from the other$(n-1)/2$ samples the $t(n+1)^{th}$ smallest ranked unit and select from the remaining sample the median for the sample for actual measurement. The cycle may be repeated r times if needed to get nr units from PRSS.

The Burr XII distribution was introduced in the literature by Burr (1942) and received more attention by the researchers due to its wide applications in different fields including the area of reliability and life testing. The probability density function (PDF) and cumulative distribution function(CDF) of Burr XII take the following forms

$$f(x;c,b) = \mathrm{b}cx^{c-1}(1+x^c)^{-(b+1)}, x > 0, c > 0, b > 0, \tag{1}$$

and

$$F(x;c,b) = 1 - (1+x^c)^{-b}, \ \ x > 0, c > 0, b > 0, \tag{2}$$

where c and b are shape parameters.

Let $Y, X_1, X_2, \ldots, X_{\mathcal{K}}$ be independent, $G(y)$ be the continuous distribution function of Y and $F(x)$ be the common continuous distribution function of $X_1, X_2, \ldots, X_{\mathcal{K}}$. Then the reliability in a multicomponent stress-strength model developed by Bhattacharyya and Johnson (1974) is given by

$$\mathrm{R}_{\mathcal{S},\mathcal{K}} = \mathrm{P[at\ least}\ \mathcal{S}\ \mathrm{of\ the}\ (\mathrm{X}_1, \mathrm{X}_2, \ldots, X_{\mathcal{K}})\mathrm{exceed\ Y]} = \sum_{i=\mathcal{S}}^{\mathcal{K}} \binom{\mathcal{K}}{i} \int_{-\infty}^{\infty} [1-F(y)]^i \, [F(y)]^{\mathcal{K}-i} dG(y). \tag{3}$$

Pandey andUddin (1991) introduced the derivation of a convenient expression for the system reliability and obtained inferences about the reliability in a multicomponent stress-strength system when both stress and strength are independently identically distributed Burr random variables. Uddin et al. (1993) provided maximum likelihood and Bayesian approaches for system reliability under the assumption that both stress and strength are independently identically Burr random variables. Rao et al. (2014) studied the multicomponent stress-strength reliability for two-parameter Burr XII distribution when both of stress and strength variables follow the same population. They obtained the asymptotic distribution and confidence interval using maximum likelihood method

Recently, estimation problem of $R = P(Y < X)$ based on RSS take the attention of many authors. Sengupta and Mukhati (2008) derived and compared reliability estimator of R based on RSS with SRS. They proved that the unbiased estimator based on RSS data has a smaller variance compared with the unbiased estimator based on SRS, even when the ranking of RSS are imperfect.Muttlak et al. (2010) considered the problem of estimating $R = P(Y < X)$, where X and Y are independently exponentially distributed random variables with different scale parameters using ranked set sampling data. Hussian (2014) discussed the estimation problem of stress- strength model for generalized inverted exponential distribution based on RSS and SRS. Maximumlikelihood method is used to estimate R using both approaches. Hassan et al. (2014) introduced the estimation of $R = P[Y < X]$ when Y and X are two independent Burr type XII distributions with common known shape parameter c. They obtained maximum likelihood estimator of $R = P(Y < X)$ based on ranked set sampling data and compared the new estimator based on RSS with known estimator based on SRS data in terms of their bias, mean square error and efficiency. They concluded that the results of estimators based on RSS are more efficient than the corresponding SRS using simulated data.Hassan et al. (2015) introduced the estimation problem of $R = P(Y < X)$ when $X$and Y are independently distributed Burr XII random variables based on different sampling schemes. They derived maximum likelihood estimators (MLEs) of R using SRS, RSS, MRSS, ERSS and PRSS techniques.

To our knowledge, in the literature, there were no studies that had been performed about stress- strength problem incorporating multicomponent systems based on ranked set sampling technique. Therefore, the main objective in this article is to derive the reliability estimators for Burr type XII distribution based on RSS, MRSS, ERSS and PRSStechniques. Simulation study is performed to compare different estimators.

The rest of the article is organized as follows. In Section 2, MLE of $R_{\mathcal{S},\mathcal{K}}$ is discussed in case of SRS data. In Section 3, 4, 5 and 6, MLEs of $R_{\mathcal{S},\mathcal{K}}$based on RSS, MRSS, ERSS and PRSS will be obtained respectively. Numerical study is presented in Section 7. Finally conclusions are presented in Section 8.

II. Maximum Likelihood Estimator of $R_{\mathcal{S},\mathcal{K}}$Based on SRS data

Let $X \sim Burr(c,b)$and $Y \sim Burr(c,a)$are independently distributed with unknown shape parameters b, a and common known shape parameter c respectively. Rao et al. (2014) derived the reliability in a multicomponent stress-strength model $R_{\mathcal{S},\mathcal{K}}$for two-parameter Burr XII distributions as the following:

$$R_{\mathcal{S},\mathcal{K}} = \sum_{i=\mathcal{S}}^{\mathcal{K}} \binom{\mathcal{K}}{i} \int_0^\infty a\, cy^{c-1}(1+y^c)^{-(a+1)}[(1+y^c)^{-b}]^i[1-(1+y^c)^{-b}]^{\mathcal{K}-i}dy$$

$$R_{\mathcal{S},\mathcal{K}} = \sum_{i=\mathcal{S}}^{\mathcal{K}} \binom{\mathcal{K}}{i} \omega\, B(\omega+i, \mathcal{K}-i+1).$$

After the simplification $R_{\mathcal{S},\mathcal{K}}$ will be

$$R_{\mathcal{S},\mathcal{K}} = \omega \sum_{i=\mathcal{S}}^{\mathcal{K}} \frac{\mathcal{K}!}{i!}\left[\prod_{j=i}^{\mathcal{K}}(\omega+j)\right]^{-1}, \tag{4}$$

where$\mathcal{K}$ and i are integers and $\omega = \frac{a}{b}$.

To derive the MLEs of b and a, let $X_1, \dots, X_n; Y_1, \dots, Y_m$be two random samples of size *n*, *m* respectively on strength, stress variables each following Burr XIIwith shapeparameters b, a and common shape parameter c. Then the log-likelihood function of the observed SRS data denoted by l is

$$l = plnb + qlna + (p+q)lnc + (c-1)\left[\sum_{i=1}^{p} lnx_i + \sum_{j=1}^{q} lny_j\right] - (b+1)$$
$$\times \sum_{i=1}^{p} ln(1+x_i^c) - (a+1)\sum_{j=1}^{q} ln\left(1+y_j^c\right). \tag{5}$$

MLEs of b and a, say $\hat{b}_{MLE}$ and $\hat{a}_{MLE}$ when c is known can be obtained by differentiating Equation (5) and equating by zero. Then $\hat{b}_{MLE}$ and $\hat{a}_{MLE}$ will be obtained as follows

$\hat{b}_{MLE} = \frac{p}{\sum_{i=1}^{p} ln(1+x_i^c)}and\hat{a}_{MLE} = \frac{q}{\sum_{j=1}^{q} ln\left(1+y_j^c\right)}$.(6)

MLE of$R_{\mathcal{S},\mathcal{K}}$ denoted by $\hat{R}_{\mathcal{S},\mathcal{K}}$, is obtained by substitute$\hat{\omega}_{MLE} = \frac{\hat{a}_{MLE}}{\hat{b}_{MLE}}$ in Equation 4 as follows:

$$\hat{R}_{\mathcal{S},\mathcal{K}} = \hat{\omega}_{MLE} \sum_{i=\mathcal{S}}^{\mathcal{K}} \frac{\mathcal{K}!}{i!}\left[\prod_{j=i}^{\mathcal{K}}(\hat{\omega}_{MLE}+j)\right]^{-1}.$$

III. Maximum Likelihood Estimator of $R_{\mathcal{S},\mathcal{K}}$Based on RSS data

In this section, MLE of the reliability in a multicomponent stress-strength model $R_{\mathcal{S},\mathcal{K}}$ based on RSS data will be derived. To compute the MLE of $R_{\mathcal{S},\mathcal{K}}$, the MLEs of b and a must be computed firstly. Let $\{X_{i(i)s},\ i = 1,2,\dots n; s=1,2,\dots,r\}$be the set of independent RSS with sample size $p=nr$, where n isthe set size and r is the number of cycles with PDF given by:

$f_i\left(x_{i(i)s}\right) = \frac{n!}{(i-1)!(n-i)!}\left[F\left(x_{i(i)s}\right)\right]^{i-1}\left[1-F\left(x_{i(i)s}\right)\right]^{n-i} f\left(x_{i(i)s}\right), \quad x_{i(i)s} > 0.$ (7).

Suppose $\{X_{1(1)s}, X_{2(2)s}, \dots, X_{n(n)s}; s=1,\dots,r\}$ denotethe ranked set sample of size $p = nr$ from Burr (c,b). Then by substituting PDF (1) and CDF (2) in PDF (7), the likelihood function, denoted by L_{RSS}, is given by:

$$L_{RSS} = \prod_{s=1}^{r}\prod_{i=1}^{n} \frac{n!}{(i-1)!\,(n-i)!} bcx_{i(i)s}^{c-1}\left(1+x_{i(i)s}^c\right)^{-[b(n-i+1)+1]}\left(1-\left(1+x_{i(i)s}^c\right)^{-b}\right)^{i-1}.$$

The log-likelihood function denoted by l_{RSS} will be as follows:

$$l_{RSS} = \xi + plnb + \sum_{s=1}^{r}\left[\sum_{i=1}^{n}(c-1)ln\, x_{i(i)s} - \sum_{i=1}^{n}[b(n-i+1)+1]ln\left(1+x_{i(i)s}^c\right)\right.$$
$$\left.+\sum_{i=1}^{n}(i-1)\, ln\left(1-(1+x_{i(i)s}^c)^{-b}\right)\right],$$

where ξ is a constant. The first partial derivative of log-likelihood function with respect to b is given by:

$\frac{\partial l_{RSS}}{\partial b} = \frac{p}{\tilde{b}} - \sum_{s=1}^{r}\sum_{i=1}^{n}[(n-i+1)\ln(1+x_{i(i)s}^{c}) - (i-1)\frac{\ln(1+x_{i(i)s}^{c})}{(1+x_{i(i)s}^{c})^{\tilde{b}}-1}] = 0.$ (8)

By similar way, let $\{Y_{1(1)s}, Y_{2(2)s}, \ldots, Y_{m(m)s}; s = 1, \ldots, r\}$denote the ranked set sample of size $q = mr$ from Burr (c, a), and applying a similar procedure described above the first partial derivative of log-likelihood function with respect to a is given as follows

$$\frac{\partial l_{RSS}}{\partial a} = \frac{q}{\tilde{a}} - \sum_{s=1}^{r}\sum_{j=1}^{m}[(m-j+1)\ln(1+y_{j(j)s}^{c}) - (j-1)\frac{\ln(1+y_{j(j)s}^{c})}{(1+y_{j(j)s}^{c})^{\tilde{a}}-1}] = 0. \quad (9)$$

Clearly, from Equations (8) and (9) it is not easy to obtain a closed form solution to get $\tilde{b}$and $\tilde{a}$. Therefore, an iterative technique must be applied to solve these equations numerically. The MLE estimator $\tilde{R}_{\mathcal{S},\mathcal{K}}$ of $R_{\mathcal{S},\mathcal{K}}$ using RSS is obtained by substituting $\tilde{b}$ and $\tilde{a}$ in Equation (4). Then $\tilde{R}_{\mathcal{S},\mathcal{K}}$ will be obtained as follows:

$\tilde{R}_{\mathcal{S},\mathcal{K}} = \tilde{\omega}_{MLE}\sum_{i=\mathcal{S}}^{\mathcal{K}}\frac{\mathcal{K}!}{i!}\left[\prod_{j=i}^{\mathcal{K}}(\tilde{\omega}_{MLE}+j)\right]^{-1}$, where$\tilde{\omega}_{MLE} = \frac{\tilde{a}}{\tilde{b}}$.(10)

IV. Maximum Likelihood Method Based on MRSS data

In the following subsections, the MLEs of $R_{\mathcal{S},\mathcal{K}}$ using the MRSS technique will be derived based on two cases, the first case for odd set size and the second case for even set size.

4.1. MLE of $R_{\mathcal{S},\mathcal{K}}$ Based on MRSS with Odd Set Size

Let $\{X_{i(g)s}, i = 1,2,\ldots,n; s = 1,2,\ldots,r\}$ be a MRSS for odd set size where $g = \frac{n+1}{2}$. Then by direct substitution in Equation (7), the PDF of $(g)^{th}$order statistics when n is odd is given as follows:

$$f_g(x_{i(g)s}) = \frac{n!}{[(g-1)!]^2}f(x_{i(g)s})[F(x_{i(g)s})]^{g-1}[1-F(x_{i(g)s})]^{g-1},\ x_{i(g)s} > 0. \quad (11)$$

Let $X_{1(g)s}, \ldots, X_{n(g)s}$ is a MRSS from Burr (c, b) with sample size $p = nr$, where n is the set size, r is the number of cycles. Then, using Equation (11) the PDF of $X_{i(g)s}$will be as follows:

$f_g(x_{i(g)s}) = \frac{n!}{[(g-1)!]^2}bcx_{i(g)s}^{c-1}[1+x_{i(g)s}^{c}]^{-(bg+1)}\left[1-(1+x_{i(g)s}^{c})^{-b}\right]^{g-1},\ x_{i(g)s} > 0.$ (12)

The likelihood function of the MRSS in case of odd set size denoted by L_{odd}^{*} is given as follows:

$$L_{odd}^{*} = \prod_{s=1}^{r}\prod_{i=1}^{n}\frac{n!}{[(g-1)!]^2}bcx_{i(g)s}^{c-1}\left[1+x_{i(g)s}^{c}\right]^{-(bg+1)}\left[1-(1+x_{i(g)s}^{c})^{-b}\right]^{g-1}.$$

The log-likelihood function of L_{odd}^{*} denoted by l_{odd}^{*} will be given as follows:

$$l_{odd}^{*} = \xi_1 + pln b + \sum_{s=1}^{r}\left[(c-1)\sum_{i=1}^{n}ln x_{i(g)s} - (bg+1)\sum_{i=1}^{n}\ln(1+x_{i(g)s}^{c}) + (g-1)\times\sum_{i=1}^{n}\ln\left[1-(1+x_{i(g)s}^{c})^{-b}\right]\right],$$

whereξ_1 is a constant. The first partial derivative of log-likelihood function with respect to b is given by:

$\frac{\partial l_{odd}^{*}}{\partial b} = \frac{p}{b_1^{*}} - \sum_{s=1}^{r}\sum_{i=1}^{n}[g\ln(1+x_{i(g)s}^{c}) - (g-1)\frac{\ln(1+x_{i(g)s}^{c})}{(1+x_{i(g)s}^{c})^{b_1^{*}}-1}] = 0.$ (13)

Similarly, let $Y_{1(h)s}, \ldots, Y_{m(h)s}$ is a MRSS from Burr (c, a) with sample size $q = mr$, where m is the set size, r is the number of cycles and $h = \frac{m+1}{2}$. Then, a_1^{*} will be obtained using a similar procedure of b_1^{*} as follows

$\frac{\partial l_{odd}^{*}}{\partial a} = \frac{q}{a_1^{*}} - \sum_{s=1}^{r}\sum_{j=1}^{m}[h\ln(1+y_{j(h)s}^{c}) - (h-1)\frac{\ln(1+y_{j(h)s}^{c})}{(1+y_{j(h)s}^{c})^{a_1^{*}}-1}] = 0.$(14)

MLEs of b and a, say, b_1^{*} and a_1^{*}, respectively, are obtained as a solution of non-linear Equations (13) and (14) using iterative technique. The MLE of $R_{\mathcal{S},\mathcal{K}}$ denoted by R_{odd}^{*} using MRSS is obtained by substituting b_1^{*} anda_1^{*} in Equation (4). Then R_{odd}^{*} will be obtained as follows:

$$R_{odd}^{*} = \omega_{odd}^{*} \sum_{i=\mathcal{S}}^{\mathcal{K}} \frac{\mathcal{K}!}{i!}\left[\prod_{j=i}^{\mathcal{K}}(\omega_{odd}^{*} + j)\right]^{-1}, \ \omega_{odd}^{*} = \frac{a_1^{*}}{b_1^{*}}.$$

4.2. MLE of $R_{\mathcal{S},\mathcal{K}}$ Based on MRSS with Even Set Size

In this subsection, MLE of $R_{\mathcal{S},\mathcal{K}}$ using MRSS in case of even set size will be obtained. Let the set $\{\{X_{i(u)s}, i = 1,\dots,u; s = 1,\dots,r\} \cup \{X_{i(u+1)s}, i = u+1,\dots,n; s = 1,2,\dots,r\}\}$, be a MRSS for even set size where $u = n/2$. Then $X_{i(u)s}$ and $X_{i(u+1)s}$ are the $(u)^{th}$ and the $(u+1)^{th}$ smallest units from the i^{th} set of the s^{th} cycle. By direct substitution in Equation (7) the PDFs of $(u)^{th}$ and $(u+1)^{th}$ order statistics when n is even are given as follows:

$$f_u(x_{i(u)s}) = \frac{n!}{(u-1)!\,(u)!} f(x_{i(u)s})\left[F(x_{i(u)s})\right]^{u-1}\left[1 - F(x_{i(u)s})\right]^{u}, \quad x_{i(u)s} > 0, \tag{15}$$

and,

$$f_{u+1}(x_{i(u+1)s}) = \frac{n!}{(u)!\,(u-1)!} f(x_{i(u+1)s})\left[F(x_{i(u+1)s})\right]^{u}\left[1 - F(x_{i(u+1)s})\right]^{u-1}, x_{i(u+1)s} > 0. \tag{16}$$

Let the set $\{X_{1(u)s},\dots,X_{n(u+1)s}; s = 1,2,\dots,r\}$ be a MRSS drawn from Burr (c,b) with even set size, where $u = n/2$. Then,by direct substitution into Equations (15) and (16), the PDFs of $(u)^{th}$ and $(u+1)^{th}$ order statistics when n is even are given as follows:

$$f_u(x_{i(u)s}) = \frac{n!}{(u-1)!\,(u)!} bcx_{i(u)s}^{c-1}\left[1 + x_{i(u)s}^{c}\right]^{-(b(u+1)+1)}\left[1 - \left(1 + x_{i(u)s}^{c}\right)^{-b}\right]^{u-1}, x_{i(u)s} > 0,$$

and

$$f_{u+1}(x_{i(u+1)s}) = \frac{n!}{(u)!\,(u-1)!} bcx_{i(u+1)s}^{c-1}\left[1 + x_{i(u+1)s}^{c}\right]^{-(bu+1)}\left[1 - \left(1 + x_{i(u+1)s}^{c}\right)^{-b}\right]^{u}, \ x_{i(u+1)s} > 0.$$

The Likelihood function denoted by L_{even}^{*} is given as follows:

$$L_{even}^{*} = \prod_{s=1}^{r}\left[\prod_{i=1}^{u} f_u(x_{i(u)s}) \prod_{i=u+1}^{n} f_{u+1}(x_{i(u+1)s})\right].$$

Then, log- likelihood function of L_{even}^{*} denoted by l_{even}^{*} will be as follows:

$$\begin{aligned} l_{even}^{*} &= \xi_2 + plnb + (c-1)\sum_{s=1}^{r}\left[\sum_{i=1}^{u} lnx_{i(u)s} + \sum_{i=u+1}^{n} lnx_{i(u+1)s}\right] - \sum_{s=1}^{r}[(b(u+1)+1) \\ &\times \sum_{i=1}^{u} ln\left[1 + x_{i(u)s}^{c}\right] + (bu+1)\sum_{i=u+1}^{n} ln\left[1 + x_{i(u+1)s}^{c}\right]] \\ &+ \sum_{s=1}^{r}\left[(u-1)\sum_{i=1}^{u} ln\left[1 - \left(1 + x_{i(u)s}^{c}\right)^{-b}\right] + u\sum_{i=u+1}^{n} ln\left[1 - \left(1 + x_{i(u+1)s}^{c}\right)^{-b}\right]\right], \end{aligned}$$

where ξ_2 is a constant. The first partial derivative of log-likelihood function with respect to b is given by:

$$\begin{aligned} \frac{\partial l_{even}^{*}}{\partial b} &= \frac{p}{b_2^{*}} - \sum_{s=1}^{r}\left[(u+1)\sum_{i=1}^{u} ln\left[1 + x_{i(u)s}^{c}\right] + u\sum_{i=u+1}^{n} ln\left[1 + x_{i(u+1)s}^{c}\right]\right. \\ &+ \sum_{s=1}^{r}[(u-1)\sum_{i=1}^{u}\frac{ln\left(1 + x_{i(u)s}^{c}\right)}{\left(1 + x_{i(u)s}^{c}\right)^{b_2^{*}} - 1} + u\sum_{i=u+1}^{n}\frac{ln\left(1 + x_{i(u+1)s}^{c}\right)}{\left(1 + x_{i(u+1)s}^{c}\right)^{b_2^{*}} - 1}] = 0. \end{aligned} \tag{17}$$

Let the set $\{\{Y_{1(v)s},\dots,Y_{v(v)s}; s = 1,2,\dots,r\} \cup \{Y_{v+1(v+1)s},\dots,Y_{m(v+1)s}; s = 1,2,\dots,r\}\}$, be a MRSS drawn from Burr (c,b) with even set size where $v = m/2$. Then by using similar procedure, a_2^{*} will be obtained using MRSS in case of even set size as follows:

$$\begin{aligned} \frac{\partial l_{even}^{*}}{\partial a} &= \frac{q}{a_2^{*}} - \sum_{s=1}^{r}[(v+1)\sum_{j=1}^{v} ln\left[1 + y_{j(v)s}^{c}\right] + v\sum_{j=v+1}^{m} ln\left[1 + y_{j(v+1)s}^{c}\right] \\ &+ \sum_{s=1}^{r}[(v-1)\sum_{j=1}^{v}\frac{ln\left(1 + y_{j(v)s}^{c}\right)}{\left(1 + y_{j(v)s}^{c}\right)^{a_2^{*}} - 1} + \sum_{j=v+1}^{m}\frac{vln\left(1 + y_{j(v+1)s}^{c}\right)}{\left(1 + y_{j(v+1)s}^{c}\right)^{a_2^{*}} - 1}] = 0. \end{aligned} \tag{18}$$

Solving Equations (17) and (18) iteratively to get the MLEs of b and $a$for MRSS data in case of even set size. The MLE of $R_{\mathcal{S},\mathcal{K}}$ denoted by R^*_{even} using MRSS is obtained by substituting the MLEs of b and a denoted by b^*_2 and a^*_2 in Equation (4). Then R^*_{even} will be as follows:

$$R^*_{even} = \omega^*_{even} \sum_{i=\mathcal{S}}^{\mathcal{K}} \frac{\mathcal{K}!}{i!} \left[\prod_{j=i}^{k} (\omega^*_{even} + j) \right]^{-1}, \omega^*_{even} = \frac{a^*_2}{b^*_2}.$$

V. Maximum Likelihood Method Based on ERSS data

In the following subsections, MLE of $R_{\mathcal{S},\mathcal{K}}$ is derived under ERSS technique for both odd and even set sizes.

5.1 MLE of $R_{\mathcal{S},\mathcal{K}}$ Based on ERSS with Odd Set Size

To obtain MLE of $R_{\mathcal{S},\mathcal{K}}$ based on ERSS in case of odd set size, let$\{X_{i(1)s}, i = 1,..,g-1; s = 1,..,r\}$ and $\{X_{i(n)s}, i = g,\dots,n-1; s = 1,2,\dots,r\}$are the smallest and largest order statistics from Burr (c,b), where n is the set size, r is the number of cycles. Therefore, the PDFs of $X_{i(1)s}$ and $X_{i(n)s}$ are given as follows:

$$f_1(x_{i(1)s}) = nbc\, x^{c-1}_{i(1)s} \left(1 + x^c_{i(1)s}\right)^{-(bn+1)}, x_{i(1)s} > 0, \tag{19}$$

and

$$f_n(x_{i(n)s}) = nbc\, x^{c-1}_{i(n)s} \left(1 + x^c_{i(n)s}\right)^{-(b+1)} \left[1 - \left(1 + x^c_{i(n)s}\right)^{-b}\right]^{n-1}, x_{i(n)s} > 0. \tag{20}$$

let$\{X_{i(g)s}, s = 1,2,\dots,r\}$ is the g^{th} order statistics from Burr (c,b), where the PDF of g^{th} order statistics is obtained in (12). The likelihood function of observed sample based on ERSS in case of odd set size, denoted byL^{**}_{odd} , is given as follows:

$$L^{**}_{odd} = \prod_{s=1}^{r} \left[f_g(x_{n(g)s}) \prod_{i=1}^{g-1} f_1(x_{i(1)s}) \prod_{i=g}^{n-1} f_n(x_{i(n)s}) \right].$$

The log-likelihood function denoted by l^{**}_{odd} is given as follows:

$$l^{**}_{odd} = \xi_3 + p\ln b + (c-1) \sum_{s=1}^{r} [\sum_{i=1}^{g-1} \ln\, x_{i(1)s} + \sum_{i=g}^{n-1} \ln x_{i(n)s}] - \sum_{s=1}^{r} [(bn+1)$$
$$\times \sum_{i=1}^{g-1} \ln\left(1 + x^c_{i(1)s}\right) + (b+1) \sum_{i=g}^{n-1} \ln\left(1 + x^c_{i(n)s}\right) - (n-1) \sum_{i=g}^{n-1} \ln\left[1 - \left(1 + x^c_{i(n)s}\right)^{-b}\right]]$$
$$+ \sum_{s=1}^{r} [(c-1)\ln x_{i(g)s} - (bg+1)\ln\left(1 + x^c_{i(g)s}\right) + (g-1) \ln\left[1 - \left(1 + x^c_{i(g)s}\right)^{-b}\right]],$$

whereξ_3 is a constant. The first partial derivative of log-likelihood function with respect to b is given by:

$$\frac{\partial l^{**}_{odd}}{\partial b} = \frac{p}{b^{**}_1} - \sum_{s=1}^{r} [n \sum_{i=1}^{g-1} \ln\left(1 + x^c_{i(1)s}\right) + \sum_{i=g}^{n-1} \ln\left(1 + x^c_{i(n)s}\right) + g\ln\left(1 + x^c_{i(g)s}\right)]$$
$$+ \sum_{s=1}^{r} [\sum_{i=g}^{n-1} \frac{(n-1)\ln\left(1 + x^c_{i(n)s}\right)}{\left(1 + x^c_{i(n)s}\right)^{b^{**}_1} - 1} + \frac{(g-1)\ln\left(1 + x^c_{i(g)s}\right)}{\left(1 + x^c_{i(g)s}\right)^{b^{**}_1} - 1}] = 0. \tag{21}$$

Similarly, let$\{Y_{j(1)s}, j = 1,..,h-1; s = 1,2,\dots r\}$, $\{Y_{j(m)s}, j = h,\dots,m-1; s = 1,2,\dots,r\}$are the smallest and largest order statistics from Burr (c,a), where m is the set size, r is the number of cycles. let $\{Y_{j(h)s}, s = 1,\dots,r\}$ is the h^{th} order statistics from Burr (c,a). Then, a^{**}_1 will be obtained using the above similar procedure as follows:

$$\frac{\partial l^{**}_{odd}}{\partial a} = \frac{q}{a^{**}_1} - \sum_{s=1}^{r} [m \sum_{j=1}^{h-1} \ln\left(1 + y^c_{j(1)s}\right) + \sum_{j=h}^{m-1} \ln\left(1 + y^c_{j(m)s}\right) + h\ln\left(1 + y^c_{j(h)s}\right)]$$
$$+ \sum_{s=1}^{r} [\sum_{j=h}^{m-1} (m-1) \frac{\ln\left(1 + y^c_{j(m)s}\right)}{\left(1 + y^c_{j(m)s}\right)^{a^{**}_1} - 1} + (h-1) \frac{\ln\left(1 + y^c_{j(h)s}\right)}{\left(1 + y^c_{j(h)s}\right)^{a^{**}_1} - 1}] = 0. \tag{22}$$

From Equations (21) and (22) MLEs of b and a denoted by b_1^{**} and a_1^{**} are obtained by using iterative technique. Then R_{odd}^{**} will be obtained by substituting b_1^{**} and a_1^{**} in Equation (4).

5.2 MLE of $R_{\mathcal{S},\mathcal{K}}$ Based on ERSS with Even Set Size

To obtain MLE of $R_{\mathcal{S},\mathcal{K}}$ based on ERSS in case of odd set size, let $\{X_{i(1)s}, i = 1,2,\dots,u; s = 1,2,\dots,r\}$ and $\{X_{i(n)s}, i = u+1,\dots,n; s = 1,2,\dots,r\}$ are the smallest and largest order statistics from Burr (c,b) with PDFs (19) and (20).

The likelihood function of observed sample using ERSS in case of even set size denoted by L_{even}^{**} is given as follows:

$$L_{even}^{**} = \prod_{s=1}^{r}\left[\prod_{i=1}^{u} f_1(x_{i(1)s}) \prod_{i=u+1}^{n} f_n(x_{i(n)s})\right].$$

Then the log-likelihood function of L_{even}^{**} denoted by l_{even}^{**} will be as follows:

$$l_{even}^{**} = \xi_4 + p\ln b + (c-1)\sum_{s=1}^{r}[\sum_{i=1}^{u} \ln x_{i(1)s} + \sum_{i=u+1}^{n} \ln x_{i(n)s}] - \sum_{s=1}^{r}[\,(bn+1)$$
$$\times \sum_{i=1}^{u} \ln\left(1 + x_{i(1)s}^{c}\right) + (b+1)\sum_{i=u+1}^{n} \ln\left(1 + x_{i(n)s}^{c}\right) - \sum_{i=u+1}^{n} (n-1)\ln\left[1 - \left(1 + x_{i(n)s}^{c}\right)^{-b}\right]\Big], \qquad (23)$$

where ξ_4 is a constant. The first partial derivative of log-likelihood function with respect to b is given by:

$$\frac{\partial l_{even}^{**}}{\partial b} = \frac{p}{b_2^{**}} - \sum_{s=1}^{r}[\,n\sum_{i=1}^{u} \ln\left(1 + x_{i(1)s}^{c}\right) + \sum_{i=u+1}^{n} \ln\left(1 + x_{i(n)s}^{c}\right) - \sum_{i=u+1}^{n} \frac{(n-1)\ln\left(1 + x_{i(n)s}^{c}\right)}{\left(1 + x_{i(n)s}^{c}\right)^{b_2^{**}} - 1}] = 0. \qquad (24)$$

Similarly, let $\{Y_{1(1)s},\dots,Y_{v(1)s}; s = 1,2,\dots,r\}$ and $\{Y_{v+1(m)s},\dots,Y_{m(m)s}; s = 1,2,\dots,r\}$ are ERSS from Burr (c,a) with sample size $q = mr$, where m is the set size, r is the number of cycles. Then, a_2^{**} will be obtained by using a similar procedure as follows

$$\frac{\partial l_{even}^{**}}{\partial a} = \frac{q}{a_2^{**}} - \sum_{s=1}^{r}[\,m\sum_{j=1}^{v} \ln\left(1 + y_{j(1)s}^{c}\right) + \sum_{j=v+1}^{m} \ln\left(1 + y_{j(m)s}^{c}\right) - \sum_{j=v+1}^{m} \frac{(m-1)\ln\left(1 + y_{j(m)s}^{c}\right)}{\left(1 + y_{j(m)s}^{c}\right)^{a_2^{**}} - 1}]$$
$$= 0. \qquad (25)$$

An iterative technique must be applied to obtain estimates of b and a, denoted by b_2^{**} and a_2^{**}, respectively, by solving Equations (24) and (25). The MLE of $R_{\mathcal{S},\mathcal{K}}$ denoted by R_{even}^{**} using ERSS is obtained by direct substitution b_2^{**} and a_2^{**} in Equation (4). Then R_{even}^{**} will be obtained as follows:

$$R_{even}^{**} = \omega_{even}^{**}\sum_{i=\mathcal{S}}^{\mathcal{K}} \frac{\mathcal{K}!}{i!}\left[\prod_{j=i}^{\mathcal{K}}(\omega_{even}^{**} + j)\right]^{-1},$$

where $\omega_{even}^{**} = \frac{a_2^{**}}{b_2^{**}}$.

VI. Maximum Likelihood Method Based on PRSS data

In this section, MLE of $R_{\mathcal{S},\mathcal{K}}$ is derived under PRSS technique depending on odd and even set sizes. The two cases are separately considered in the following subsections.

6.1 MLE of $R_{\mathcal{S},\mathcal{K}}$ Based on PRSS Data with Odd Set Size

Let n_1 and n_2 be the nearest integer values of $s(n+1)$ and $t(n+1)$ respectively, where $0 < s \le 0.5$ and $t = 1 - s$. Then for odd set size, the PRSS is the set: $\{X_{i(n_1)s}, i = 1,\dots,g-1; s = 1,\dots,r\} \cup \{X_{i(n_2)s}, i = g,\dots,n-1; s = 1,\dots,r\} \cup \{X_{n(g)s}, s = 1,2,\dots,r\}$, where $g = \frac{n+1}{2}$. Then using Equation (7) the PDFs of n_1th and n_2th order statistics are given as follows:

$$f_{n_1}(x_{i(n_1)s}) = \frac{n!}{(n-n_1)!(n_1-1)!}\left[F(x_{i(n_1)s})\right]^{n_1-1}\left[1 - F(x_{i(n_1)s})\right]^{n-n_1} f(x_{i(n_1)s}), \quad (26)$$

and

$$f_{n_2}(x_{i(n_2)s}) = \frac{n!}{(n-n_2)!(n_2-1)!}\left[F(x_{i(n_2)s})\right]^{n_2-1}\left[1 - F(x_{i(n_2)s})\right]^{n-n_2} f(x_{i(n_2)s}). \quad (27)$$

The PDF of g^{th} order statistics was defined in Equation (11).

Let $X_{1(n_1)s},\dots,X_{g-1(n_1)s}, X_{g(n_2)s},\dots,X_{n-1(n_2)s}, X_{n(g)s}$ is a PRSS from Burr (c,b) with sample size $p = nr$, where n is the set size and r is the number of cycles. Then using Equations (26) and (27) the PDFs of $X_{i(n_1)s}$ and $X_{i(n_2)s}$ will be as follows:

$$f_{n_1}\left(x_{i(n_1)s}\right) = \frac{n!}{(n-n_1)!(n_1-1)!} bcx_{i(n_1)s}^{c-1}\left(1+x_{i(n_1)s}^{c}\right)^{-[b(n-n_1+1)+1]}\left(1-\left(1+x_{i(n_1)s}^{c}\right)^{-b}\right)^{n_1-1},\ x_{i(n_1)s} > 0, \quad (28)$$

and

$$f_{n_2}\left(x_{i(n_2)s}\right) = \frac{n!}{(n-n_2)!(n_2-1)!} bcx_{i(n_2)s}^{c-1}\left(1+x_{i(n_2)s}^{c}\right)^{-[b(n-n_2+1)+1]}\left(1-\left(1+x_{i(n_2)s}^{c}\right)^{-b}\right)^{n_2-1}.\ x_{i(n_2)s} > 0. \quad (29)$$

While the PDF of $X_{i(g)s}$ was defined in Equation (12).

The likelihood function of the observed PRSS in case of odd set size denoted by $\grave{L}_{odd}$ is given by:

$$\grave{L}_{odd} = \prod_{s=1}^{r}\left[f_g\left(X_{n(g)s}\right)\right]\left[\prod_{i=1}^{g-1} f_{n_1}\left(x_{i(n_1)s}\right)\prod_{i=g}^{n-1} f_{n_2}\left(x_{i(n_2)s}\right)\right].$$

The log-likelihood function of $\grave{L}_{odd}$ denoted by $\grave{l}_{odd}$ will be as follows:

$$\grave{l}_{odd} = \xi_5 + p\ln b + (c-1)\sum_{s=1}^{r}\left[\sum_{i=1}^{g-1} ln\left(x_{i(n_1)s}\right) + \sum_{i=g}^{n-1} ln\left(x_{i(n_2)s}\right) + \ln\left(x_{n(g)s}\right)\right]$$

$$-\sum_{s=1}^{r}[(b(n-n_1+1)+1)\sum_{i=1}^{g-1} ln\left(1+x_{i(n_1)s}^{c}\right) + (b(n-n_2+1)+1)\sum_{i=g}^{n-1} ln\left(1+x_{i(n_2)s}^{c}\right)$$

$$+(bg+1)ln\left(1+x_{n(g)s}^{c}\right)] + \sum_{s=1}^{r}[(n_1-1)\times\sum_{i=1}^{g-1} ln\left(1-\left(1+x_{i(n_1)s}^{c}\right)^{-b}\right) + (n_2-1)$$

$$\times\sum_{i=g}^{n-1} ln\left(1-\left(1+x_{i(n_2)s}^{c}\right)^{-b}\right) + (g-1)ln\left(1-\left(1+x_{n(g)s}^{c}\right)^{-b}\right)],$$

where ξ_5 is a constant. The first partial derivative of b is given by:

$$\frac{\partial\grave{l}_{odd}}{\partial b} = \frac{p}{\grave{b}_1} - \sum_{s=1}^{r}[\,(n-n_1+1)\sum_{i=1}^{g-1} ln\left(1+x_{i(n_1)s}^{c}\right) + (n-n_2+1)\sum_{i=g}^{n-1} ln\left(1+x_{i(n_2)s}^{c}\right) + gln\left(1+x_{n(g)s}^{c}\right)$$

$$-(n_1-1)\sum_{i=1}^{g-1}\frac{ln\left(1+x_{i(n_1)s}^{c}\right)}{\left(1+x_{i(n_1)s}^{c}\right)^{\grave{b}_1}-1} - \sum_{i=g}^{n-1}\frac{(n_2-1)ln\left(1+x_{i(n_2)s}^{c}\right)}{\left(1+x_{i(n_2)s}^{c}\right)^{\grave{b}_1}-1} - (g-1)\frac{ln\left(1+x_{n(g)s}^{c}\right)}{\left(1+x_{n(g)s}^{c}\right)^{\grave{b}_1}-1}] = 0. \quad (30)$$

Similarly, let m_1 and m_2 be the nearest integer values of $s(m+1)$ and $t(m+1)$ respectively. Then for odd set size, the PRSS is the set $\{Y_{j(m_1)s}, j = 1,..,h-1; s = 1,\dots,r\} \cup \{Y_{j(m_2)s}, j = h,..,m-1; s = 1,\dots,r\} \cup \{Y_{m(h)s}, s = 1,2,\dots,r\}$, where $h = m + 1/2$. Then, $\grave{a}_1$ will be obtained using a similar procedure as follows:

$$\frac{\partial\grave{l}_{odd}}{\partial a} = \frac{q}{\grave{a}_1} - \sum_{s=1}^{r}[\,(m-m_1+1)\sum_{j=1}^{h-1} ln\left(1+y_{j(m_1)s}^{c}\right) + (m-m_2+1)\sum_{j=h}^{m-1} ln\left(1+y_{j(m_2)s}^{c}\right)$$

$$+hln\left(1+y_{m(h)s}^{c}\right)] - (m_1-1)\sum_{j=1}^{h-1}\frac{\ln\left(1+y_{j(m_1)s}^{c}\right)}{\left(1+y_{j(m_1)s}^{c}\right)^{\grave{a}_1}-1} - (m_2-1)\sum_{j=h}^{m-1}\frac{ln\left(1+y_{j(m_2)s}^{c}\right)}{\left(1+y_{j(m_2)s}^{c}\right)^{\grave{a}_1}-1}$$

$$-(h-1)\frac{ln\left(1+y_{m(h)s}^{c}\right)}{\left(1+y_{m(h)s}^{c}\right)^{\grave{a}_1}-1} = 0. \quad (31)$$

Obviously, there is no closed form solution to non-linear Equations (30) and (31). Therefore, an appropriate numerical technique is applied. Then $\grave{R}_{odd}$ will be obtained by substituting $\grave{b}_1$ and $\grave{a}_1$ in Equation (4).

6.2 MLE of $R_{\mathcal{S},\mathcal{K}}$ Based on PRSS Data with Even Set Size

In this subsection, MLE of $R_{\mathcal{S},\mathcal{K}}$ is derived under PRSS approach depending on even set size. Let n_1, n_2, s and t be defined as in previous subsection (6.1), then for even set size, the PRSS is the set $\{X_{i(n_1)s}, i =$ *1,...,u ; s =1 ,...,r ∪ Xin2s,i=u+1,...,n;s=1,...,r*, where, *u=n2*. Then the PDFs of *n1th* and *n2th* order statistics are given in Equations (26) and (27). Let $X_{1(n_1)s}, \dots, X_{n(n_2)s}$ is PRSS from Burr (c, b) with sample size $p = nr$, where n is the set size, r is the number of cycles. The PDFs of $X_{i(n_1)s}$ and $X_{i(n_2)s}$ are givenin Equations (28) and (29).

The likelihood function of observed PRSS in case of even set size denoted by $\grave{L}_{even}$ is given by:

$$\grave{L}_{even}=\prod_{s=1}^{r}\left[\prod_{i=1}^{u} f_{n_1}(x_{i(n_1)s})\prod_{i=u+1}^{n} f_{n_2}(x_{i(n_2)s})\right].$$

The log-likelihood function of PRSS data in case of even set size denoted by $\grave{l}_{even}$ is given by:

$$\grave{l}_{even}=\xi_6+plnb+(c-1)\sum_{s=1}^{r}\left[\sum_{i=1}^{u} ln\, x_{i(n_1)s}+\sum_{i=u+1}^{n} ln\, x_{i(n_2)s}\right]+\sum_{s=1}^{r}\left[(n_1-1)\sum_{i=1}^{u} ln\right.$$
$$\left[1-\left(1+x_{i(n_1)s}^{c}\right)^{-b}\right]+(n_2-1)\sum_{i=u+1}^{n} \ln\left[1-\left(1+x_{i(n_2)s}^{c}\right)^{-b}\right]-[b(n-n_1+1)+1]$$
$$\sum_{i=1}^{u} ln\left(1+x_{i(n_1)s}^{c}\right)-[b(n-n_2+1)+1]\sum_{i=u+1}^{n} ln\left(1+x_{i(n_2)s}^{c}\right),$$

where ξ_6 is a constant. The first partial derivative of log-likelihood function with respect to b is given by:

$$\frac{\partial\grave{l}_{even}}{\partial b}=\frac{p}{\grave{b}_2}+\sum_{s=1}^{r}[\,(n_1-1)\sum_{i=1}^{u}\frac{ln\left(1+x_{i(n_1)s}^{c}\right)}{\left(1+x_{i(n_1)s}^{c}\right)^{\grave{b}_2}-1}+(n_2-1)\sum_{i=u+1}^{n}\frac{ln\left(1+x_{i(n_2)s}^{c}\right)}{\left(1+x_{i(n_2)s}^{c}\right)^{\grave{b}_2}-1}$$
$$-[(n-n_1+1)\times\sum_{i=1}^{u} ln\left(1+x_{i(n_1)s}^{c}\right)+(n-n_2+1)\sum_{i=u+1}^{n} ln\ \left(1+x_{i(n_2)s}^{c}\right)]]=0. \quad (32)$$

Similarly, let the set $\{Y_{j(m_1)s}, j=1,..,v; s=1,\dots,r\}\cup\{Y_{j(m_2)s}, j=v+1,..,m; s=1,\dots,r\}$ is the PRSS for even set size, where $v=m/2$. Let $Y_{1(m_1)s},\dots,Y_{v(m_1)s},Y_{v+1(m_2)s},\dots,Y_{m(m_2)s}$ is a PRSS from Burr (c,a) with sample size $q=mr$, where m is the set size, r is the number of cycles. Then,$\grave{a}_2$ will be obtained using a similar procedure as follows:

$$\frac{\partial\grave{l}_{even}}{\partial a}=\frac{q}{\grave{a}_2}+\sum_{s=1}^{r}\left[(m_1-1)\sum_{j=1}^{v}\frac{ln\left(1+y_{j(m_1)s}^{c}\right)}{\left(1+y_{j(m_1)s}^{c}\right)^{\grave{a}_2}-1}+(m_2-1)\sum_{j=v+1}^{m}\frac{ln\left(1+y_{j(m_2)s}^{c}\right)}{\left(1+y_{j(m_2)s}^{c}\right)^{\grave{a}_2}-1}\right]$$
$$-\left[(m-m_1+1)\sum_{j=1}^{v} ln\left(1+y_{j(m_1)s}^{c}\right)+(m-m_2+1)\sum_{j=v+1}^{m} ln\left(1+y_{j(m_2)s}^{c}\right)\right]=0. \quad (33)$$

To obtain $\grave{b}_2$ and $\grave{a}_2$, an appropriate numerical technique can be used to solve analytically Equations (32) and (33). Then $\grave{R}_{even}$ will be obtained by substituting $\grave{b}_2$ and $\grave{a}_2$ in Equation (4).

VI. Numerical Study

In this study, the reliability estimators in multicomponent stress strength model following Burr type XII distributions based on RSS and some of its modifications will be compared, paying attention to the odd and even set size namely, $\tilde{R}_{MLE}, R_{odd}^{*}, R_{odd}^{**}, \grave{R}_{odd}, R_{even}^{*}, R_{even}^{**}$ and$\grave{R}_{even}$, to the estimator $\hat{R}_{MLE}$based on SRS data. The comparison will be performed using MSEs and efficiencies. The efficiencies of these estimators will be reported with respect to $\hat{R}_{MLE}$. The efficiencies of these estimators depend on a and b through the ratio $\omega=\frac{a}{b}$.

1000 random samples of sizes $p=nr$ and $q=mr$ from the stress and strength populations are generated respectively, where $n=m=3,4,5,6,7,8$ and $r=10$ is the number of cycles. The ratio ω is selected as 0.1, 0.5, 1, 2 and 6.MSEs and efficiencies will be reported in Tables $1-8$ and represented through Figures $1-4$. From these tables many observations can be made on the performance of different estimators.

1- MSEs of $\hat{R}_{MLE}$ based on SRS are greater than MSEs of all estimators$\tilde{R}_{MLE}, R_{odd}^{*}, R_{odd}^{**}, \grave{R}_{odd}, R_{even}^{*}$, R_{even}^{**} and $\grave{R}_{even}$based on RSS, MRSS, ERSS and PRSS data respectively.
2- In case of odd set size, MSEs of $\grave{R}_{odd}$based on PRSS data at s = 0.40 are the smallest one in all cases except at [ω =2, $(n,m)=(5,5)$ and $(\mathcal{S},\mathcal{K})=(1,4)$].
3- In case of even set size, MSEs of $\grave{R}_{even}$based on PRSS data at s = 0.30are the smallest one in almost all cases.
4- In all cases, MSEs of all estimators $\hat{R}_{MLE}$,$\tilde{R}_{MLE}, R_{odd}^{*}, R_{odd}^{**}, \grave{R}_{odd}, R_{even}^{*}, R_{even}^{**}$ and$\grave{R}_{even}$decrease as the set sizes increase at the same value of ω.
5- In case of $(\mathcal{S},\mathcal{K})=(2,4)$, MSEs of all estimators$\hat{R}_{MLE}, \tilde{R}_{MLE}, R_{odd}^{*}, R_{odd}^{**}, \grave{R}_{odd}, R_{even}^{*}, R_{even}^{**}$ and $\grave{R}_{even}$ increase as the value of ω increases up to $\omega=1$, then MSEs decrease as the value of ω increases in all cases. (See for example Figure (1)).

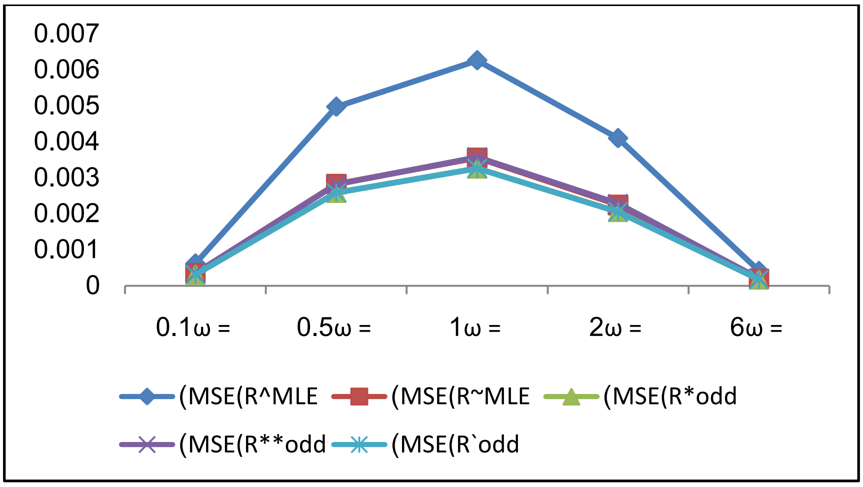

Figure (1): MSEs of the estimators $\hat{R}_{MLE}, \tilde{R}_{MLE}, R^{*}_{odd}, R^{**}_{odd}$ and $\grave{R}_{odd}$ for odd set size at $(\mathcal{S},\mathcal{K}) = (2,4)$ where $(n,m) = (3,3)$ and s = 0.40.

6. In case of parallel system at $(\mathcal{S},\mathcal{K}) = (1,4)$ MSEs of all estimators $\hat{R}_{MLE}, \tilde{R}_{MLE}, R^{*}_{odd}, R^{**}_{odd}, \grave{R}_{odd}, R^{*}_{even}, R^{**}_{even}$ and $\grave{R}_{even}$ increase as the value of ω increases up to $\omega = 0.5$ then MSEs decrease as the value of ω increases in all cases. (See for example Figure (2)).

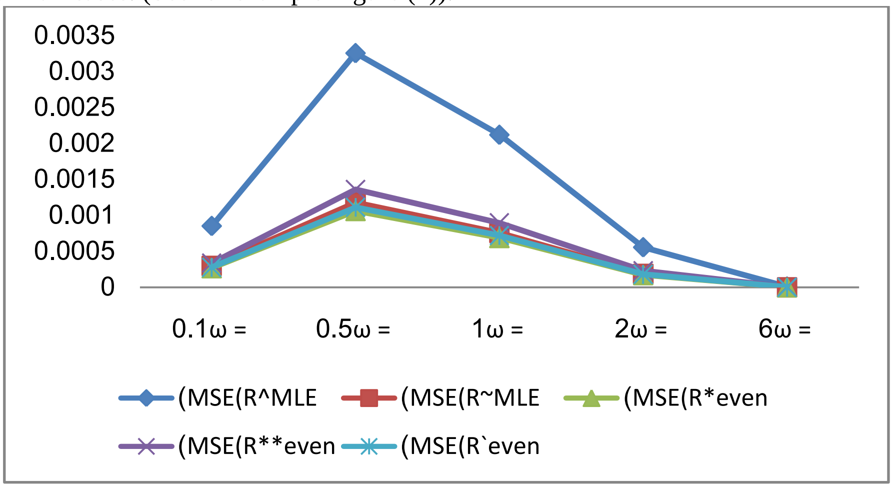

Figure (2): MSEs of the estimators $\hat{R}_{MLE}, \tilde{R}_{MLE}, R^{*}_{even}, R^{**}_{even}$ and $\grave{R}_{even}$ for even set size at $(\mathcal{S},\mathcal{K}) = (1,4)$ where $(n,m) = (6,6)$ and s = 0.30.

7. In case of series system at $(\mathcal{S},\mathcal{K}) = (4,4)$ MSEs of all estimators $\hat{R}_{MLE}, \tilde{R}_{MLE}, R^{*}_{odd}, R^{**}_{odd}, \grave{R}_{odd}, R^{*}_{even}, R^{**}_{even}$ and $\grave{R}_{even}$ increase as the value of ω increases, in all cases. (See for example Figure (3)).

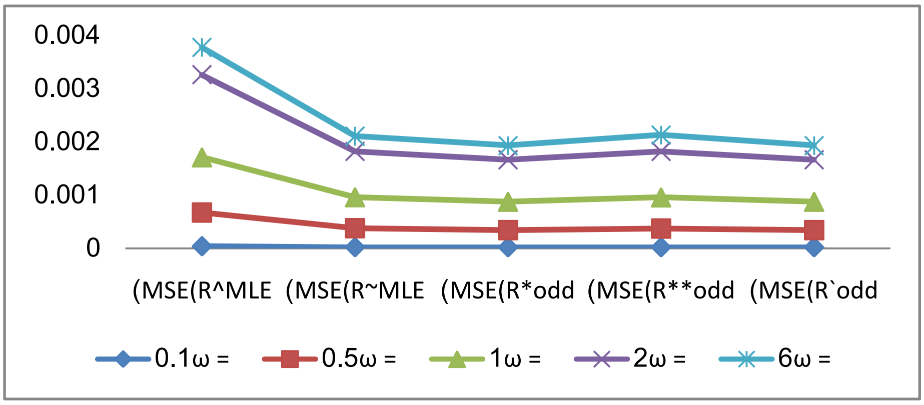

Figure (3): MSEs of the estimators $\hat{R}_{MLE}, \tilde{R}_{MLE}, R^{*}_{odd}, R^{**}_{odd}$ and $\grave{R}_{odd}$ for odd set size at $(\mathcal{S},\mathcal{K}) = (4,4)$ where $(n,m) = (3,3)$ and s = 0.40.

8. For the special case $\mathcal{S} = \mathcal{K} = 1$, MSEs of all estimators$\hat{R}_{MLE}$, $\tilde{R}_{MLE}$, R^*_{odd}, R^{**}_{odd}, $\grave{R}_{odd}$, R^*_{even}, R^{**}_{even} and $\grave{R}_{even}$ increase as the value of ω increases up to $\omega = 1$, then MSEsdecrease as the value of ω increases in all cases. (See for example Figure (4)).

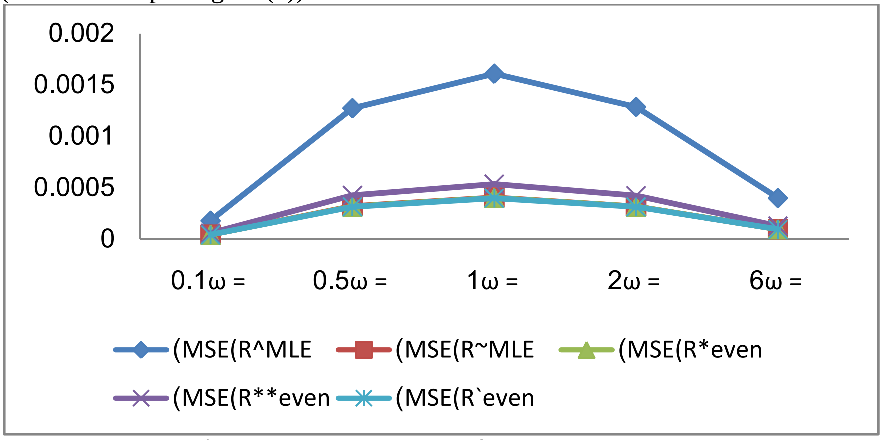

Figure (4): MSEs of the estimators $\hat{R}_{MLE}$, $\tilde{R}_{MLE}$, R^*_{even} R^{**}_{even} and $\grave{R}_{even}$ for even set size at $(\mathcal{S},\mathcal{K}) = (1,1)$ where $(n, m) = (6,6)$ and s = 0.30.

9. In case of odd set size, the efficiencies of $\grave{R}_{odd}$ based on PRSS data at s = 0.40 are the largest comparing to the other estimators $\hat{R}_{MLE}$, $\tilde{R}_{MLE}$, R^*_{odd}, R^{**}_{odd}.
10. In case of even set size, the efficiencies of $\grave{R}_{even}$ based on PRSS data at s = 0.30 are the largest comparing to the other estimators $\hat{R}_{MLE}$, $\tilde{R}_{MLE}$, R^*_{even}, R^{**}_{even}.
11. In all cases, efficiencies of all estimators $\tilde{R}_{MLE}$, R^*_{odd}, R^{**}_{odd}, $\grave{R}_{odd}$, R^*_{even}, R^{**}_{even} and $\grave{R}_{even}$ increase as the set sizes increase at the same value of ω.
12. In almost all cases, in case of $(\mathcal{S},\mathcal{K}) = (2,4)$efficiencies of all estimators $\hat{R}_{MLE}$, $\tilde{R}_{MLE}$, R^*_{odd}, R^{**}_{odd}, $\grave{R}_{odd}$ R^*_{even}, R^{**}_{even} and$\grave{R}_{even}$ decrease as the value of ω increases up to $\omega = 1$, then the efficiencies increase as the value of ω increases.
13. In almost all cases, in case of parallel system at $(\mathcal{S},\mathcal{K}) = (1,4)$ the efficiencies of all estimators $\hat{R}_{MLE}$, $\tilde{R}_{MLE}$, R^*_{odd}, R^{**}_{odd}, $\grave{R}_{odd}$, R^*_{even}, R^{**}_{even} and $\grave{R}_{even}$ decrease as the value of ω increases up to $\omega = 0.5$,then the efficienciesincrease as the value of ω increases.
14. In almost all cases, in case of series system at $(\mathcal{S},\mathcal{K}) = (4,4)$ the efficiencies of all estimators $\hat{R}_{MLE}$, $\tilde{R}_{MLE}$, R^*_{odd}, R^{**}_{odd}, $\grave{R}_{odd}$, R^*_{even}, R^{**}_{even} and$\grave{R}_{even}$ decrease as the value of ω increases.
15. For the special case $\mathcal{S} = \mathcal{K} = 1$, the efficiencies of all estimators $\hat{R}_{MLE}$, $\tilde{R}_{MLE}$, R^*_{odd}, R^{**}_{odd}, $\grave{R}_{odd}$, R^*_{even}, R^{**}_{even} and$\grave{R}_{even}$ decrease as the value of ω increases up to $\omega = 1$, then the efficiencies increase as the value of ω increases in almost all cases.

VII. Conclusions

The goal of this article is to estimate the multicomponent stress-strength model using maximum likelihood method of estimation based on RSS, MRSS, ERSS, and PRSS techniques when the strengths and the stress follow Burr XII distributions.

From the simulation results, the estimators of $R_{\mathcal{S},\mathcal{K}}$ based on PRSS, MRSS, RSS and ERSS, respectively dominate the corresponding estimator based on SRS data in all cases.

In all cases, MSEs of all estimators based on RSS technique and its modifications are smaller than the corresponding MSEs depend on SRS approach. In addition, MSEs of all estimators decrease as the set sizes increase.

It is clear from the simulation study that the efficiency of all estimators' increases as the set size increases. In case of odd set size, MLE of $R_{\mathcal{S},\mathcal{K}}$based on PRSS at s = 0.40 is moreefficient than the other sampling approaches, namely MRSS, RSS, ERSS and SRS respectively. In case of even set size, MLE of $R_{\mathcal{S},\mathcal{K}}$based on PRSS at s = 0.30 is more efficient than the other different sampling techniques.

The simulation study indicates that in order to estimate the reliability in multicomponent stress-strength model for Burr XII distribution the PRSS technique is preferable than the other sampling techniques.

Table (1): MSEs and efficiencies of the estimators $\hat{R}_{MLE}, \tilde{R}_{MLE}, R^{*}_{odd}, R^{**}_{odd}$ and $\grave{R}_{odd}$ for odd set sizes at $(\mathcal{S},\mathcal{K}) = (2,4)$.

ω	(n,m)	$\hat{R}_{MLE}$	$\tilde{R}_{MLE}$	R^{*}_{odd}	R^{**}_{odd}	$\grave{R}_{odd}$		
						20%	30%	40%
0.1	(3,3)	0.000618	0.000342 (1.860)	0.000306 (2.093)	0.000340 (1.817)	0.000340 (1.817)	0.000340 (1.817)	0.000306 (2.093)
	(5,5)	0.000372	0.000138 (2.682)	0.000133 (2.781)	0.000137 (2.702)	0.000137 (2.702)	0.000129 (2.880)	0.000129 (2.880)
	(7,7)	0.000257	0.000067 (3.838)	0.000066 (3.903)	0.000072 (3.580)	0.000065 (3.936)	0.000056 (3.936)	0.000061 (4.176)
0.5	(3,3)	0.004973	0.002826 (1.760)	0.002585 (1.924)	0.002823 (1.761)	0.002823 (1.761)	0.002823 (1.761)	0.002585 (1.924)
	(5,5)	0.003033	0.001184 (2.564)	0.001119 (2.709)	0.001182 (2.566)	0.001182 (2.566)	0.001111 (2.730)	0.001111 (2.730)
	(7,7)	0.002137	0.000579 (3.690)	0.000572 (3.735)	0.000621 (3.441)	0.000563 (3.793)	0.000563 (3.793)	0.000259 (4.036)
1	(3,3)	0.006262	0.003552 (1.763)	0.003259 (1.922)	0.003576 (1.751)	0.003576 (1.751)	0.003576 (1.751)	0.003259 (1.922)
	(5,5)	0.003770	0.001507 (2.501)	0.001452 (2.596)	0.001511 (2.495)	0.001511 (2.495)	0.001422 (2.650)	0.001422 (2.650)
	(7,7)	0.002684	0.000739 (3.636)	0.000734 (3.653)	0.000793 (3.382)	0.000718 (3.738)	0.000718 (3.738)	0.000672 (3.990)
2	(3,3)	0.004100	0.002243 (1.828)	0.002054 (1.996)	0.002282 (1.797)	0.002282 (1.797)	0.002282 (1.797)	0.002054 (1.996)
	(5,5)	0.002346	0.000944 (2.484)	0.000896 (2.616)	0.000949 (2.471)	0.000949 (2.471)	0.000895 (2.619)	0.000895 (2.619)
	(7,7)	0.001680	0.000460 (3.650)	0.000461 (3.643)	0.000494 (3.400)	0.000445 (3.772)	0.000445 (3.772)	0.000415 (4.043)
6	(3,3)	0.000411	0.000193 (2.124)	0.000175 (2.343)	0.000200 (2.058)	0.000200 (2.058)	0.000200 (2.058)	0.000175 (2.343)
	(5,5)	0.000197	0.000075 (2.629)	0.000072 (2.745)	0.000075 (2.616)	0.000075 (2.616)	0.000071 (2.762)	0.000071 (2.762)
	(7,7)	0.000140	0.000035 (3.961)	0.000036 (3.895)	0.000037 (3.702)	0.000033 (4.137)	0.000033 (4.137)	0.000031 (4.466)

The efficiencies are reported in brackets in the table.

Table (2): MSEs and efficiencies of the estimators $\hat{R}_{MLE}, \tilde{R}_{MLE}, R^{*}_{odd}, R^{**}_{odd}$ and $\grave{R}_{odd}$ for odd set sizes at $(\mathcal{S},\mathcal{K}) = (1,4)$.

ω	(n,m)	$\hat{R}_{MLE}$	$\tilde{R}_{MLE}$	R^{*}_{odd}	R^{**}_{odd}	$\grave{R}_{odd}$		
						20%	30%	40%
0.1	(3,3)	0.001746	0.000976 (1.789)	0.000887 (1.966)	0.000951 (1.834)	0.000951 (1.834)	0.000951 (1.834)	0.000887 (1.966)
	(5,5)	0.001057	0.000399 (2.646)	0.000380 (2.781)	0.000397 (2.662)	0.000397 (2.662)	0.000372 (2.836)	0.000372 (2.836)
	(7,7)	0.000735	0.000194 (3.793)	0.000191 (3.853)	0.000208 (3.537)	0.000189 (3.892)	0.000189 (3.892)	0.000178 (4.132)
0.5	(3,3)	0.006516	0.003702 (1.760)	0.003396 (1.919)	0.003725 (1.750)	0.003725 (1.750)	0.003725 (1.750)	0.003396 (1.919)
	(5,5)	0.003932	0.001571 (2.503)	0.001488 (2.642)	0.001575 (2.497)	0.001575 (2.497)	0.001482 (2.653)	0.001482 (2.653)
	(7,7)	0.002798	0.000770 (3.630)	0.000766 (3.656)	0.000823 (3.383)	0.000748 (3.738)	0.000748 (3.738)	0.000701 (3.989)
1	(3,3)	0.004410	0.002419 (1.823)	0.002217 (1.989)	0.002462 (1.791)	0.002462 (1.791)	0.002462 (1.791)	0.002217 (1.989)
	(5,5)	0.002531	0.001021 (2.480)	0.000983 (2.574)	0.001026 (2.467)	0.001026 (2.467)	0.000968 (2.615)	0.000968 (2.615)
	(7,7)	0.001814	0.000498 (3.642)	0.000499 (3.637)	0.000534 (3.393)	0.000497 (3.644)	0.000497 (3.644)	0.000449 (4.034)

The efficiencies are reported in brackets in the table.

Continued Table (2)

ω	(n,m)	$\hat{R}_{MLE}$	$\tilde{R}_{MLE}$	R^{*}_{odd}	R^{**}_{odd}	$\grave{R}_{odd}$		
						20%	30%	40%
2	(3,3)	0.001289	0.000644 (2.000)	0.000586 (2.197)	0.000663 (1.943)	0.000663 (1.943)	0.000663 (1.943)	0.000586 (2.197)
	(5,5)	0.000662	0.000259 (2.555)	0.000245 (2.700)	0.000261 (2.539)	0.000261 (2.539)	0.000247 (2.684)	0.000247 (2.684)
	(7,7)	0.000473	0.000123 (3.819)	0.000125 (3.771)	0.000132 (3.562)	0.000119 (3.973)	0.000119 (3.973)	0.000110 (4.280)
6	(3,3)	0.000025	0.000009 (2.570)	0.000008 (2.858)	0.000010 (2.480)	0.000010 (2.480)	0.000010 (2.480)	0.000008 (2.858)
	(5,5)	0.000010	0.000003 (2.915)	0.000003 (3.030)	0.000003 (2.912)	0.000003 (2.912)	0.000003 (3.064)	0.000003 (3.064)
	(7,7)	0.000007	0.000001 (4.505)	0.000001 (4.378)	0.000001 (3.702)	0.000001 (4.781)	0.000001 (4.781)	0.000001 (5.190)

The efficiencies are reported in brackets in the table.

Table (3): MSEs and efficiencies of the estimators $\hat{R}_{MLE}, \tilde{R}_{MLE}, R^{*}_{odd}, R^{**}_{odd}$ and $\grave{R}_{odd}$ for odd set sizes at $(\mathcal{S},\mathcal{K}) = (4,4)$.

ω	(n,m)	$\hat{R}_{MLE}$	$\tilde{R}_{MLE}$	R^{*}_{odd}	R^{**}_{odd}	$\grave{R}_{odd}$		
						20%	30%	40%
0.1	(3,3)	0.000040	0.000022 (1.822)	0.000020 (1.966)	0.000021 (1.867)	0.000021 (1.867)	0.000021 (1.867)	0.000020 (1.966)
	(5,5)	0.000024	0.000009 (2.713)	0.000008 (2.816)	0.000008 (2.738)	0.000008 (2.738)	0.000008 (2.919)	0.000008 (2.919)
	(7,7)	0.000016	0.000004 (3.878)	0.000004 (3.948)	0.000004 (3.618)	0.000004 (3.977)	0.000004 (3.977)	0.000003 (4.216)
0.5	(3,3)	0.000674	0.000375 (1.795)	0.000341 (1.974)	0.000373 (1.805)	0.000373 (1.805)	0.000373 (1.805)	0.000341 (1.974)
	(5,5)	0.000407	0.000153 (2.658)	0.000144 (2.818)	0.000152 (2.675)	0.000152 (2.675)	0.000143 (2.850)	0.000143 (2.850)
	(7,7)	0.000283	0.000074 (3.807)	0.000073 (3.869)	0.000079 (3.551)	0.000074 (3.783)	0.000074 (3.783)	0.000068 (4.146)
1	(3,3)	0.001715	0.000965 (1.776)	0.000880 (1.948)	0.000962 (1.783)	0.000962 (1.783)	0.000962 (1.783)	0.000880 (1.948)
	(5,5)	0.001042	0.000399 (2.611)	0.000383 (2.718)	0.000397 (2.622)	0.000397 (2.622)	0.000373 (2.792)	0.000373 (2.792)
	(7,7)	0.000729	0.000194 (3.749)	0.000191 (3.803)	0.000208 (3.497)	0.000198 (3.682)	0.000198 (3.682)	0.000178 (4.091)
2	(3,3)	0.003261	0.001823 (1.764)	0.001667 (1.928)	0.001823 (1.764)	0.001823 (1.764)	0.001823 (1.764)	0.001667 (1.928)
	(5,5)	0.001945	0.000763 (2.559)	0.000726 (2.689)	0.0000762 (2.562)	0.000762 (2.562)	0.000717 (2.726)	0.000717 (2.726)
	(7,7)	0.001378	0.000373 (3.689)	0.000369 (3.731)	0.000400 (3.439)	0.000363 (3.793)	0.000363 (3.793)	0.000341 (4.038)
6	(3,3)	0.003775	0.002110 (1.789)	0.001933 (1.953)	0.002130 (1.773)	0.002130 (1.773)	0.002130 (1.773)	0.001933 (1.953)
	(5,5)	0.002232	0.000889 (2.508)	0.000847 (2.635)	0.000935 (2.385)	0.000935 (2.385)	0.000840 (2.656)	0.000840 (2.656)
	(7,7)	0.001589	0.000435 (3.652)	0.000436 (3.873)	0.000466 (3.403)	0.000422 (3.765)	0.000422 (3.765)	0.000395 (4.023)

The efficiencies are reported in brackets in the table.

Table (4): MSEs and efficiencies of the estimators $\hat{R}_{MLE}, \tilde{R}_{MLE}, R^{*}_{odd}, R^{**}_{odd}$ and $\grave{R}_{odd}$ for odd set sizes at $(\mathcal{S}, \mathcal{K}) = (1,1)$.

ω	(n,m)	$\hat{R}_{MLE}$	$\tilde{R}_{MLE}$	R^{*}_{odd}	R^{**}_{odd}	$\grave{R}_{odd}$		
						20%	30%	40%
0.1	(3,3)	0.000476	0.000264 (1.800)	0.000240 (1.980)	0.0002634 (1.811)	0.0002634 (1.811)	0.0002634 (1.811)	0.000240 (1.982)
	(5,5)	0.0002875	0.000107 (2.670)	0.000103 (2.768)	0.0001069 (2.688)	0.0001069 (2.688)	0.0001003 (2.865)	0.0001003 (2.865)
	(7,7)	0.0001994	0.0000521 (3.822)	0.0000512 (3.886)	0.0000559 (3.565)	0.0000508 (3.921)	0.0000508 (3.921)	0.0000479 (4.161)
0.5	(3,3)	0.003216	0.001823 (1.764)	0.001667 (1.928)	0.001774 (1.812)	0.001774 (1.812)	0.001774 (1.812)	0.001667 (1.928)
	(5,5)	0.001954	0.000763 (2.559)	0.000722 (2.706)	0.0007628 (2.562)	0.0007628 (2.562)	0.000717 (2.726)	0.000717 (2.726)
	(7,7)	0.001378	0.000373 (3.689)	0.000369 (3.731)	0.0004006 (3.439)	0.0003633 (3.793)	0.0003633 (3.793)	0.000341 (4.038)
1	(3,3)	0.004047	0.002283 (1.772)	0.002092 (1.934)	0.002297 (1.762)	0.002297 (1.762)	0.002297 (1.762)	0.002092 (1.934)
	(5,5)	0.002427	0.000963 (2.519)	0.000927 (2.617)	0.000965 (2.515)	0.000965 (2.515)	0.0009081 (2.673)	0.0009081 (2.673)
	(7,7)	0.0001724	0.000471 (3.654)	0.0005062 (3.405)	0.0005062 (3.405)	0.0004581 (3.762)	0.0004581 (3.762)	0.000429 (4.014)
2	(3,3)	0.003285	0.001819 (1.806)	0.001665 (1.973)	0.001840 (1.785)	0.001840 (1.785)	0.001840 (1.785)	0.001665 (1.973)
	(5,5)	0.001917	0.000764 (2.506)	0.000727 (2.637)	0.0007675 (2.498)	0.0007675 (2.498)	0.0007232 (2.651)	0.0007232 (2.651)
	(7,7)	0.001367	0.000373 (3.659)	0.000372 (3.668)	0.0004757 (2.859)	0.000362 (3.776)	0.000362 (3.776)	0.000338 (4.038)
6	(3,3)	0.001073	0.0005701 (1.883)	0.000520 (2.060)	0.000580 (1.849)	0.000580 (1.849)	0.000580 (1.849)	0.000520 (2.060)
	(5,5)	0.0005954	0.000235 (2.524)	0.000225 (2.645)	0.0002369 (2.513)	0.0002369 (2.513)	0.0002236 (2.663)	0.0002236 (2.663)
	(7,7)	0.0004247	0.000114 (3.717)	0.000114 (3.706)	0.0001226 (3.465)	0.0001104 (3.847)	0.0001104 (3.847)	0.000103 (4.125)

The efficiencies are reported in brackets in the table.

Table (5): MSEs and efficiencies of the estimators $\hat{R}_{MLE}, \tilde{R}_{MLE}, R^{*}_{even}, R^{**}_{even}$ and $\grave{R}_{even}$ for even set sizes at $(\mathcal{S}, \mathcal{K}) = (2,4)$.

ω	(n,m)	$\hat{R}_{MLE}$	$\tilde{R}_{MLE}$	R^{*}_{even}	R^{**}_{even}	$\grave{R}_{even}$		
						20%	30%	40%
0.1	(4,4)	0.000457	0.000194 (2.352)	0.0001836 (2.493)	0.000228 (2.005)	0.000228 (2.005)	0.000172 (2.654)	0.0001836 (2.493)
	(6,6)	0.000298	0.000104 (2.884)	0.000097 (3.041)	0.000116 (2.557)	0.000116 (2.557)	0.000970 (3.076)	0.000097 (3.041)
	(8,8)	0.000233	0.000058 (4.023)	0.000057 (4.096)	0.000057 (4.034)	0.000057 (4.034)	0.000056 (4.109)	0.000057 (4.096)
0.5	(4,4)	0.003680	0.001652 (2.227)	0.001569 (2.346)	0.001916 (1.921)	0.001916 (1.921)	0.001455 (2.529)	0.001569 (2.346)
	(6,6)	0.002482	0.000891 (2.784)	0.000798 (3.107)	0.001012 (2.451)	0.001012 (2.451)	0.000835 (2.971)	0.000798 (3.107)
	(8,8)	0.001974	0.000498 (3.957)	0.000492 (4.006)	0.000504 (3.917)	0.000504 (3.917)	0.000490 (4.025)	0.000492 (4.006)
1	(4,4)	0.004554	0.002104 (2.164)	0.002009 (2.266)	0.002422 (1.880)	0.002422 (1.880)	0.001839 (2.476)	0.002009 (2.266)
	(6,6)	0.003126	0.001131 (2.763)	0.001019 (3.067)	0.001306 (2.394)	0.001306 (2.394)	0.001068 (2.928)	0.001019 (3.067)
	(8,8)	0.002515	0.000635 (3.960)	0.000630 (3.989)	0.000649 (3.872)	0.000649 (3.872)	0.000627 (4.012)	0.000630 (3.989)

The efficiencies are reported in brackets in the table.

Continued Table (5)

ω	(n,m)	$\hat{R}_{MLE}$	$\tilde{R}_{MLE}$	R^{*}_{even}	R^{**}_{even}	$\grave{R}_{even}$ 20%	30%	40%
2	(4,4)	0.002838	0.001327 (2.140)	0.001275 (2.226)	0.001517 (1.871)	0.001517 (1.871)	0.001143 (2.484)	0.001275 (2.226)
	(6,6)	0.001965	0.000701 (2.801)	0.000635 (3.091)	0.000828 (2.371)	0.000828 (2.371)	0.000667 (2.944)	0.000635 (3.091)
	(8,8)	0.001594	0.0003936 (4.055)	0.000392 (4.061)	0.000408 (3.901)	0.000408 (3.901)	0.000390 (4.085)	0.000392 (4.061)
6	(4,4)	0.000244	0.0001090 (2.236)	0.000105 (2.312)	0.000124 (1.969)	0.000124 (1.969)	0.000090 (2.698)	0.000105 (2.312)
	(6,6)	0.000165	0.000054 (3.054)	0.000049 (3.354)	0.000066 (2.476)	0.000066 (2.476)	0.000052 (3.186)	0.000049 (3.354)
	(8,8)	0.000133	0.0000298 (4.482)	0.000030 (4.0450)	0.000031 (4.191)	0.000031 (4.191)	0.000029 (4.473)	0.000030 (4.0450)

The efficiencies are reported in brackets in the table.

Table (6): MSEs and efficiencies of the estimators $\hat{R}_{MLE}$, $\tilde{R}_{MLE}$, R^{*}_{even}, R^{**}_{even} and $\grave{R}_{even}$ for even set sizes at $(\mathcal{S},\mathcal{K}) = (1,4)$.

ω	(n,m)	$\hat{R}_{MLE}$	$\tilde{R}_{MLE}$	R^{*}_{even}	R^{**}_{even}	$\grave{R}_{even}$ 20%	30%	40%
0.1	(4,4)	0.001294	0.000559 (2.315)	0.000528 (2.450)	0.000653 (1.798)	0.000653 (1.798)	0.000494 (2.616)	0.000528 (2.450)
	(6,6)	0.000852	0.000300 (2.833)	0.000268 (3.179)	0.000337 (2.526)	0.000337 (2.526)	0.000280 (3.044)	0.000268 (3.179)
	(8,8)	0.000669	0.000167 (3.999)	0.000164 (4.066)	0.000167 (3.998)	0.000167 (3.998)	0.000164 (4.081)	0.000164 (4.066)
0.5	(4,4)	0.004750	0.002193 (2.166)	0.002093 (2.270)	0.002525 (1.881)	0.002525 (1.881)	0.001918 (2.477)	0.002093 (2.270)
	(6,6)	0.003258	0.001180 (2.762)	0.001062 (3.067)	0.001360 (2.396)	0.001360 (2.396)	0.001113 (2.928)	0.001062 (3.067)
	(8,8)	0.002620	0.000662 (3.956)	0.000657 (3.987)	0.000676 (3.872)	0.000676 (3.872)	0.000653 (4.009)	0.000657 (3.987)
1	(4,4)	0.003051	0.001433 (2.136)	0.001377 (2.223)	0.001639 (1.868)	0.001639 (1.868)	0.001235 (2.475)	0.001377 (2.223)
	(6,6)	0.002121	0.000758 (2.795)	0.000687 (3.085)	0.000895 (2.368)	0.000895 (2.368)	0.000721 (2.939)	0.000687 (3.085)
	(8,8)	0.001722	0.000425 (4.047)	0.000424 (4.053)	0.000442 (3.895)	0.000442 (3.895)	0.000422 (4.077)	0.000424 (4.053)
2	(4,4)	0.000811	0.000371 (2.184)	0.000359 (2.259)	0.000422 (1.920)	0.000422 (1.920)	0.000312 (2.599)	0.000359 (2.259)
	(6,6)	0.000556	0.000189 (2.945)	0.000172 (3.236)	0.000230 (2.419)	0.000230 (2.419)	0.000181 (3.075)	0.000172 (3.236)
	(8,8)	0.000451	0.000104 (4.310)	0.0001053 (4.288)	0.000111 (4.062)	0.000111 (4.062)	0.000104 (4.0312)	0.0001053 (4.288)
6	(4,4)	0.0000128	0.000005 (2.425)	0.000052 (2.514)	0.000005 (2.149)	0.000005 (2.149)	0.000004 (3.072)	0.000052 (2.514)
	(6,6)	0.000008	0.000002 (3.462)	0.000002 (3.804)	0.000003 (2.698)	0.000003 (2.698)	0.000002 (3.610)	0.000002 (3.804)
	(8,8)	0.000006	0.000001 (5.123)	0.000001 (5.052)	0.000001 (4.670)	0.000001 (4.670)	0.000001 (5.073)	0.000001 (5.052)

The efficiencies are reported in brackets in the table.

Table (7): MSEs and efficiencies of the estimators $\hat{R}_{MLE}, \tilde{R}_{MLE}, R^{*}_{even}, R^{**}_{even}$ and $\grave{R}_{even}$ for even set sizes at $(\mathcal{S},\mathcal{K}) = (4,4)$.

ω	(n,m)	$\hat{R}_{MLE}$	$\tilde{R}_{MLE}$	R^{*}_{even}	R^{**}_{even}	$\grave{R}_{even}$		
						20%	30%	40%
0.1	(4,4)	0.000030	0.000012 (2.386)	0.000011 (2.532)	0.000014 (2.028)	0.000014 (2.028)	0.000011 (2.689)	0.000011 (2.532)
	(6,6)	0.000019	0.000006 (2.876)	0.000006 (3.239)	0.000007 (2.583)	0.000007 (2.583)	0.000006 (3.105)	0.000006 (3.239)
	(8,8)	0.000015	0.000003 (4.045)	0.000003 (4.123)	0.000003 (4.067)	0.000003 (4.067)	0.000003 (4.135)	0.000003 (4.123)
0.5	(4,4)	0.000499	0.000214 (2.327)	0.000202 (2.464)	0.000251 (1.988)	0.000251 (1.988)	0.000190 (2.628)	0.000202 (2.464)
	(6,6)	0.000328	0.000115 (2.840)	0.000102 (3.189)	0.000129 (2.536)	0.000129 (2.536)	0.000107 (3.055)	0.000102 (3.189)
	(8,8)	0.000257	0.000064 (4.007)	0.000063 (4.076)	0.000064 (4.009)	0.000064 (4.009)	0.000062 (4.090)	0.000063 (4.076)
1	(4,4)	0.001271	0.000558 (2.279)	0.000528 (2.407)	0.000650 (1.955)	0.000650 (1.955)	0.000492 (2.579)	0.000528 (2.407)
	(6,6)	0.000845	0.000300 (2.812)	0.000268 (3.148)	0.000338 (2.495)	0.000338 (2.495)	0.000280 (3.014)	0.000268 (3.148)
	(8,8)	0.000667	0.000167 (3.981)	0.000165 (4.040)	0.000168 (3.963)	0.000168 (3.963)	0.000164 (4.057)	0.000165 (4.040)
2	(4,4)	0.002372	0.001066 (2.224)	0.001013 (2.341)	0.001236 (1.919)	0.001236 (1.919)	0.000938 (2.528)	0.001013 (2.341)
	(6,6)	0.001601	0.000574 (2.785)	0.000515 (3.107)	0.000653 (2.448)	0.000653 (2.448)	0.000538 (2.971)	0.000515 (3.107)
	(8,8)	0.001275	0.000321 (2.962)	0.000317 (4.009)	0.000325 (3.916)	0.000325 (3.916)	0.000316 (4.028)	0.000317 (4.009)
6	(4,4)	0.002701	0.001245 (2.169)	0.001191 (2.268)	0.001432 (1.986)	0.001432 (1.986)	0.001084 (2.492)	0.001191 (2.268)
	(6,6)	0.001852	0.000665 (2.783)	0.000600 (3.086)	0.000772 (2.395)	0.000772 (2.395)	0.000629 (2.945)	0.000600 (3.086)
	(8,8)	0.001492	0.000373 (3.998)	0.000370 (4.023)	0.000383 (3.895)	0.000383 (3.895)	0.000368 (4.055)	0.000370 (4.023)

The efficiencies are reported in brackets in the table.

Table (8): MSEs and efficiencies of the estimators $\hat{R}_{MLE}, \tilde{R}_{MLE}, R^{*}_{even}, R^{**}_{even}$ and $\grave{R}_{even}$ for even set sizes at $(\mathcal{S},\mathcal{K}) = (1,1)$.

ω	(n,m)	$\hat{R}_{MLE}$	$\tilde{R}_{MLE}$	R^{*}_{even}	R^{**}_{even}	$\grave{R}_{even}$		
						20%	30%	40%
0.1	(4,4)	0.000352	0.000150 (2.339)	0.000142 (2.478)	0.000176 (1.997)	0.000176 (1.997)	0.000133 (2.641)	0.000142 (2.478)
	(6,6)	0.000230	0.000081 (2.848)	0.000072 (3.199)	0.000090 (2.546)	0.000090 (2.546)	0.000075 (3.065)	0.000072 (3.199)
	(8,8)	0.000180	0.000045 (4.015)	0.000044 (4.086)	0.000059 (3.020)	0.000044 (4.021)	0.000044 (4.099)	0.000044 (4.086)
0.5	(4,4)	0.002372	0.001066 (2.224)	0.001066 (2.224)	0.001236 (1.919)	0.001236 (1.919)	0.000938 (2.528)	0.001066 (2.224)
	(6,6)	0.001601	0.000574 (2.785)	0.0005747 (2.785)	0.000653 (2.448)	0.000653 (2.448)	0.000538 (2.971)	0.0005747 (2.785)
	(8,8)	0.001275	0.000321 (3.962)	0.000318 (4.009)	0.000426 (2.990)	0.000325 (3.916)	0.000316 (4.028)	0.001971 (4.009)
1	(4,4)	0.002938	0.001346 (2.182)	0.001346 (2.182)	0.001552 (1.893)	0.001552 (1.893)	0.001177 (2.496)	0.001346 (2.182)
	(6,6)	0.002006	0.007228 (2.776)	0.007228 (2.776)	0.000832 (2.410)	0.000832 (2.410)	0.000681 (2.945)	0.007228 (2.776)
	(8,8)	0.001610	0.000405 (3.974)	0.000402 (4.006)	0.000536 (3.002)	0.000413 (3.892)	0.000399 (4.028)	0.000402 (4.006)

The efficiencies are reported in brackets in the table.

Continued Table (8)

ω	(n,m)	$\hat{R}_{MLE}$	$\tilde{R}_{MLE}$	R^{*}_{even}	R^{**}_{even}	$\grave{R}_{even}$		
						20%	30%	40%
2	(4,4)	0.002321	0.001073 (2.164)	0.001027 (2.260)	0.00123 (1.885)	0.00123 (1.885)	0.000930 (2.495)	0.001073 (2.164)
	(6,6)	0.001595	0.000570 (2.794)	0.0005709 (2.794)	0.000666 (2.394)	0.000666 (2.394)	0.000540 (2.951)	0.0005709 (2.794)
	(8,8)	0.001287	0.000319 (4.022)	0.000318 (4.042)	0.000423 (3.040)	0.000329 (3.904)	0.000316 (4.064)	0.000318 (4.042)
6	(4,4)	0.000723	0.000333 (2.172)	0.000333 (2.172)	0.000381 (1.898)	0.000381 (1.898)	0.000285 (2.533)	0.000333 (2.172)
	(6,6)	0.000497	0.000174 (2.849)	0.000174 (2.849)	0.000206 (2.403)	0.000206 (2.403)	0.000165 (2.996)	0.000174 (2.849)
	(8,8)	0.0004018	0.000097 (4.126)	0.000097 (4.131)	0.000128 (3.116)	0.000101 (3.962)	0.000096 (4.154)	0.000097 (4.131)

The efficiencies are reported in brackets in the table.

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